

MATH Math 162A Review: Vector Calculus

1. Let A be a square matrix, and let

$$\sin A = A - \frac{A^3}{3!} + \frac{A^5}{5!} - \frac{A^7}{7!} + \cdots$$

Prove that

$$\frac{d \sin(At)}{dt^2} + A^2 \sin(At) = 0$$

Solution:

2. Let $V = (v_1, v_2, v_3)$ be an $\mathbb{R}^3$ -valued function of x, y, z . The $\text{curl } V$ is defined by

$$\text{curl } V = \left(\left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z} \right), \left(\frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x} \right), \left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \right) \right)$$

The divergence $\text{div } V$ is defined by

$$\text{div } V = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

Prove that

$$\text{div curl } V = 0.$$

Solution: